Equivalent transformations of trees with nullor and mirror pathological elements

Gennadiy Mayko
Broadcom corporation,
Irvine, CA USA
e-mail: gmayko@gmail.com

Vladimir Filaretov
Department of Electrical Engineering
Ulyanovsk State Technical University, Ulyanovsk, Russia
e-mail: vvfil@mail.ru

Konstantin Gorshkov
Department of Electrical Engineering
ITMO University
Saint-Petersburg, Russia
e-mail: K.Gorshkov@ulstu.ru

Abstract — **In this paper the method of circuit synthesis using transformation of trees with pathological elements is discussed. The generalization and mathematical proof of equivalent transformations of trees with mirror and/or nullor pathological elements are presented. Algorithm of generating equivalent trees with these pathological elements is also suggested. It is based on tagging of the vertices (nodes) by taking into account a type of pathological elements between these nodes and preserving these tags during transformations of trees. Illustrative example of universal filter synthesis by means of proposed algorithm is included.**

Key words: active analog circuits, circuit synthesis, current mirror, equivalent circuits, nullator, nullor, norator, pathological elements, voltage mirror.

I. Introduction

One of the effective methods of synthesis of active analog circuits is their equivalent transformations [1]–[7]. This method can be used when the original structure of the circuit has been obtained as a result of an invention or some other methods of synthesis. During this procedure, only circuit topology (interconnection of elements) is changed.

The equivalence of circuits is due to the fact that the system of Kirchhoff's equations, which describes such circuits, is identical. This is easy to achieve when ideal models of elements (especially active) are used in these equivalent circuit transformations. However, characteristics and parameters of equivalent circuits may be different and superior than of the original circuit in case of using more complex models of elements and, of course, real elements. Therefore, the method of equivalent circuit transformations is widely used at final stages of design and optimization of active analog circuits, such as filters, oscillators, amplifiers, etc. [2].

Probably the most popular methods of equivalent transformations are based on the transformation of circuits with ideal active elements in the form of nullors (a pair of pathological elements – nullator and norator) [1]–[6] presented in Fig. 1(a) and Fig. 1(b).

Fig. 1. Nullor and mirror pathological elements: nullator (a), norator (b), voltage mirror (c), current mirror (d).

This work was supported in part by the Russian Foundation for Basic Research (RFBR) under grant No. 15-07-05847; in part by Government of Russian Federation, Grant No. 074-U01.

978-1-5090-1201-5/15/$31.00 ©2015 IEEE

Nullator is a two-pole in which voltage and current is equal to 0:

$$\begin{cases} U = 0 \\ I = 0 \end{cases} \quad (1)$$

Norator is two-pole in which voltage U and current I are arbitrary and do not depend on each other.

New types of pathological elements – voltage mirror (VM) and current mirror (CM), are presented in Fig. 1(c) and Fig. 1(d) correspondingly. VM and CM are successfully used in the analysis and synthesis of electronic circuits [7]–[10]. Unlike the nullor, these elements have three poles (one of which is ground). However it is useful to use them as a two-port element (similar to nullors) and do not–show the third ground node [9].

Current I through floating nodes of VM is 0 and voltages at these nodes U_1 and U_2 are equal in magnitude but opposite in sign:

$$\begin{cases} U_1 = -U_2 \\ I = 0 \end{cases} \quad (2)$$

Current through floating terminals of CM I_1 and I_2 are equal in magnitude but opposite in sign (direction) and the voltage on these terminals U_1 and U_2 may have any value and is not dependent on the current through this element:

$$\begin{cases} I_1 = -I_2 \\ U_1 - U_2 = any \end{cases} \quad (3)$$

Active elements in the original analog circuit are replaced during equivalent transformation with their nullor models. This replacement completely breaks the description of active elements connection. These active elements are simply not visible in the nullor equivalent circuit of the original circuit. Then, during the restoration of active elements from their nullor models, it is possible to combine nullors differently and this leads to new equivalent circuits [3].

Another way of circuit transformations is equivalent transformations of trees for nullators or norators [3], [4] as it is shown on Fig. 2(a) or Fig. 2(b). In this case, after replacing active elements in the circuit with the corresponding pathological elements, these possible trees with nullators and norators may be transformed in a new set of equivalent trees. Then pathological elements in all these circuits may be paired together using combinatorial methods. All these circuits will be equivalent to each other.

The widespread usage of nullors and mirror elements in the design of electronic circuits is primarily because these pathological elements are the basis of ideal models of large number of different active elements: from basic ones (like transistors) to more complex (like operational amplifiers, current converters and mirrors, etc.) [2], [7]–[9].

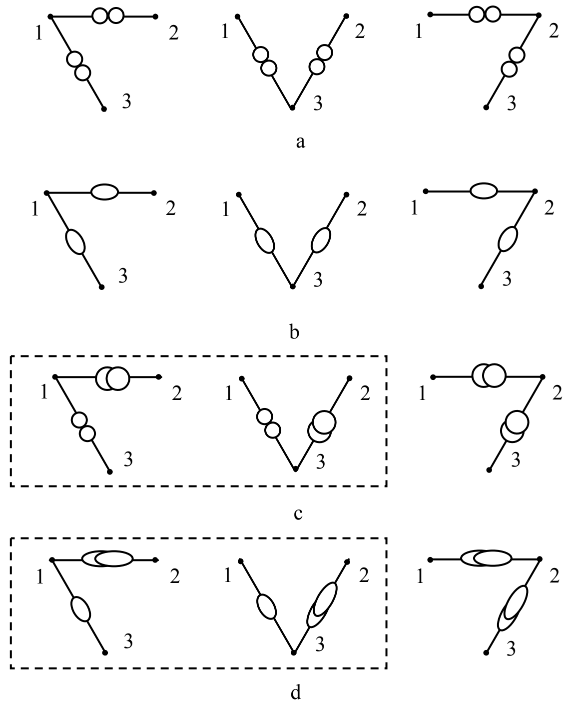

Fig. 2. Equivalent trees of pathological elements: nullators (a), norators (b), voltage mirrors and nullators (c), current mirrors and norators (d).

First the transformations for norator/nullator trees were proposed in [1] (Fig. 2(a) and Fig. 2(b)). On the Fig. 2(c) are shown all possible transformations for norators and CM proposed in [5]. Equivalent transformations for trees with norators/CM and nullator/VM shown in Fig. 2(c) and Fig. 2(d) inside dashed line are presented in [6]. All tree transformations shown in Fig. 2(d) are developed in later publication of the same authors [7]. Note that the transformations in every publication mentioned above were presented without proving.

In this paper we proposed the general approach to circuit synthesis by means of equivalent transformations of trees with all types of pathological elements. The mathematical definitions based on equations of the Kirchhoff's laws, graph theory [11] and method of equation system transformations [12] for equivalent transformations of trees with mixed pathological elements are proposed in Section II. In Section III the example of circuit synthesis by means of discussed equivalent transformations is presented.

II. Equivalent Transformations of Trees with Mixed Pathological Elements

Let us divide pathological elements into two groups. The first group consists of VM and nullators. The second group consists of CM and norators. We will consider circuits in each group of pathological elements are interconnected and form a tree. There may be several non-connected trees with the elements from the same group. Also, trees of the elements of different groups may have common nodes. Initially, we assume that there are no common (ground) nodes in each tree.

Pathological elements that form a tree designate certain relations between the voltages of corresponding nodes (for trees with elements of the first group) or the current through edges of the tree (for trees with elements of the second group). For example, voltages of all nodes are identical for any tree consisting only of nullators (they will have the same absolute value and sign). Voltages of nodes for trees with VM and possibly nullators will be different, since according to (2) VM inverts the voltage on its terminals.

A. Trees with nullators and VM

Fig. 3(a) shows the connection of two VM to a network in a way that they are forming a tree on nodes 1, 2 and 3.

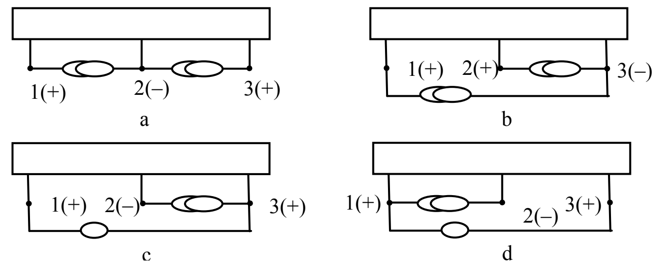

Fig. 3. Connection of the trees of VM and nullator to some circuit: original tree of VM elements (a), non-equivalent tree of VM elements (b), equivalent trees of VM and nullator (c) and (d).

For the circuit on Fig. 3(a), assuming the voltage of node 1 to be U_1 and using the properties of the VM (2), we obtain the following equations:

$$U_1 = -U_2, \tag{4}$$

$$U_2 = -U_3. \tag{5}$$

Combining (4) and (5), we will get:

$$+U_1 = -U_2 = +U_3\ . \tag{6}$$

We will now consider what will happen with the voltages of the same nodes when another tree of VM on the same nodes will be used. For example, reconnecting VM to nodes 1 and 3 (Fig. 3(b)), we obtain the following:

$$U_1 = -U_3, \tag{7}$$

$$U_2 = -U_3, \tag{8}$$

Combining (7) and (8), we will have:

$$+U_1 = +U_2 = -U_3, \tag{9}$$

Comparing (6) and (9), one can see that the various trees with VM on these same nodes, in contrast to the trees with nullators, do not lead to the equivalent circuits because of the difference in these node voltages. In order to keep the equivalence of circuits on Fig. 3(a) and Fig. 3(b), it is necessary to replace the lower VM (in Fig. 3(b)) by nullator, as shown in Fig. 3(c).

The question arises how to choose the type of the edge of the tree – VM or nullator – in order to provide these same voltages as in the original circuit? Type of the edges can be determined based on the tags in the incident nodes. If tags of these nodes are the same, the edge is replaced by a nullator, if tags are opposite, the edge is replaced by VM (Table I, 1–4).

We will illustrate this process with the example of the circuit in Fig. 3. It is known [11], that for these three nodes one can build 3 non-isomorphic trees, presented in Fig. 4.

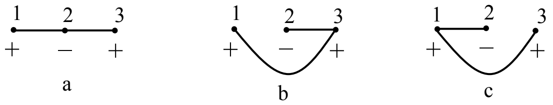

Fig. 4. All trees built on three nodes with tags from the circuit on Fig. 3(a).

Tree on Fig. 4(a) corresponds to the original circuit on Fig. 3(a). The signs of voltages for these nodes obtained from (6) are also shown on it. Nodes of other trees on Fig. 4(b) and Fig. 4(c) should be tagged by the same tags in order to ensure the circuit equivalence. Now it is possible to define the types of the edges of trees in Fig. 4(b) and Fig. 4(c) according to rules in the Table I. For example, in Fig. 4(b) between nodes 1 and 3 there should be a nullator, and the edge connecting nodes 2 and 3 should be VM. Corresponding circuit was obtained previously and is shown on Fig. 3(c). Equivalent circuit corresponding to the tree in Fig. 4(c) is shown on Fig. 3(d).

Note that equivalent circuits in Fig. 3(c) and Fig. 3(d) were presented before in [6], without specifying that they are equivalent to the circuit with two VM in Fig. 3(a).

TABLE I. Correspondence of Elements and Node Tags

N	Node tags		Elements
1	+	–	VM (CM)
2	+	+	nullator (norator)
3	–	–	nullator (norator)
4	–	+	VM (CM)
5	0	0	VM or nullator (CM or norator)

B. *Trees with norators and CM*

We will consider now the circuits with two CMs (elements of the second group) having a common node and forming a tree on the nodes 1, 2 and 3 (Fig. 5(a)).

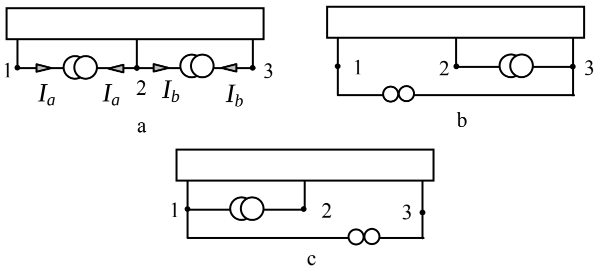

Fig. 5. Connection of trees with CM and norator elements to some circuit: original CM tree (a), equivalent CM and norator trees (b) and (c).

Equations of the first Kirchhoff's law for the nodes 1, 2 and 3 of this circuit are:

$$\begin{cases} \sum I_1 = +I_a \\ \sum I_2 = +I_a + I_b \\ \sum I_3 = +I_b \end{cases} \tag{10}$$

Left side of equations in (10) represents a sum of all the currents for the corresponding nodes, and the right side has the currents of pathological elements.

In order to eliminate the currents I_a and I_b, which uncertain values do not affect the solution of the system, we will use a method of equation system transformations, similar to proposed in [12]. Firstly we will subtract equation for node 1 from that of node 2:

$$\begin{cases} \sum I_2 - \sum I_1 = +I_b \\ \sum I_3 = +I_b \end{cases} \tag{11}$$

Subtraction operation can be shown with opposite tags for nodes 1 and 2 in the tree. Then we will subtract the second equation from the first one in (10). Again, this subtraction can be shown with opposite tags for these nodes. However, since one of the nodes (number 2) has already been assigned a tag, we should use it and assign the opposite tag to another node (number 3). Therefore, the tree of two CM can also be represented as shown on Fig. 4(a). Trees of the equivalent circuits with CM (similar to the trees with two VM) are presented in Fig. 4(b) and Fig. 4(c).

The trees that were generated for the circuits in Fig. 3(a) and Fig. 5(a) are identical. However, for the trees containing CM, these tags show not the signs of node voltages, but the operation that should be undertaken with the equations for currents of incident edges, that is, addition or subtraction of the equations by the first Kirchhoff's law for these nodes.

For example, according to Fig. 4(b) it is necessary to add equations for nodes 1 and 3 because these nodes have the same tag. Then it is necessary to subtract second equation from the result of the first operation because nodes 2 and 3 have opposite tags.

It is known [12] that operation of adding equations for some nodes corresponds to connection of the norator between these nodes. Therefore, for a tree in Fig. 4(b) we will have the circuit with pathological elements from the second group on Fig. 5(b). Similarly, Fig. 4(c) corresponds to the other connection of CM and norator as on Fig. 5(c). Rules for selecting type of edge of the pathological elements from the second group are also shown in Table I.

To assign tags to all nodes of the tree from the original circuit one can arbitrarily select tag of any node and then distribute these tags to other nodes of the tree according to the Table I, considering the type of the edge that are incident to the next node. There are two possible distributions of tags, which are inversely relative to each other. Nevertheless, selection of pathological elements for these two trees is the same because of the symmetry in rules of Table I.

Also, this procedure does not result in the ambiguity of tag assignment of these nodes in the tree. Ambiguity can arise only when there are multiple paths from one node to another, but the tree, by definition, has no loops.

C. Grounded trees with pathological elements

We will now consider the special case of the trees with VM and nullators, in which one of the nodes is connected to the ground. The voltage of this node is equal to 0 by definition. This means that using the properties of VM (2) and nullators (1), the voltages of all the other nodes will also be zero. This will be true for all trees, built on these nodes, and for any distribution of pathological elements as edges of a tree.

For example, consider the circuit with a grounded VM in Fig. 6(a). This circuit corresponds to the tree of two VMs.

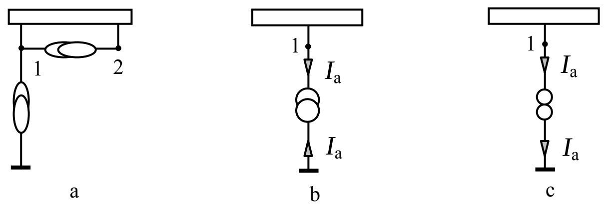

Fig. 6. Trees with VM, connected to ground node (a); single CM (b) and norator (c), connected to ground.

Since the voltage of all nodes is equal to 0, then we denote this fact on the tree (Fig. 7(a)).

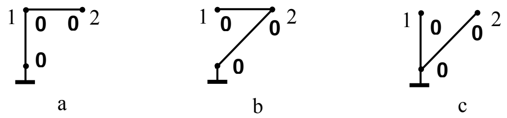

Fig. 7. All possible trees built on three nodes with these tags from circuit on Fig. 6(a).

Any edge in each of the tree in Fig. 7 can be selected as either VM or as nullator. For example, using the tree in Fig. 6(a) it is possible to generate three equivalent trees with pathological elements of the first group (Fig. 8).

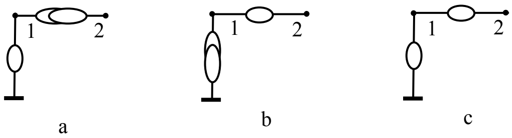

Fig. 8. Equivalent grounded trees with VM and/or nullators.

The last row of Table I corresponds to 0-tagged nodes that are incident to some VM or nullator.

Before considering grounded trees of pathological elements of the second group, we will analyze two circuits with only one grounded element – CM or norator, shown in Fig. 6(b) and Fig. 6(c), respectively. According to [9], circuits in Fig. 6(b) and Fig. 6(c) are equivalent.

Let's select the same direction of currents through these pathological elements (from the node 1). The equations of first Kirchhoff's law for node 1 in both circuits will be:

$$\sum I_1 = +I_a, \tag{12}$$

where I_a is a current of the pathological element.

From this equation, it turns out that CM connected to the ground behaves in exactly the same way as the norator also connected to the ground. Therefore, for both these circuits we can use a well-known rule for excluding grounded norators [12] (the exclusion of corresponding node equation).

Now we will add to this grounded pathological element others so they form a grounded tree. We will still assume that the node 1 is connected to a grounded pathological element, and the positive direction of the current is from the node 1. Then the Kirchhoff's equation for node 1 will be as follows:

$$\sum I_1 = +I_a + \left(\sum_i^{N_n} I_i + \sum_j^{N_{CM}} I_j\right), \tag{13}$$

where I_i are currents of all N_n norators and I_j are currents of all N_{CM} current mirrors, connected to the node 1. Since the tree of pathological elements does not have loops, then all currents I_i and I_j will also be included into these equations for some other (N_n+N_{CM}) nodes of the circuit. Now we will exclude grounded pathological element by removing the equation for node 1. It is clear that new system of equations will still contain some currents of pathological elements, incidental to the node 1, and all these currents will be presented in the equations only once.

Recursively applying the above exclusion rule for equations of remaining pathological elements in the tree we will finally obtain a system of equations in which all these equations corresponding to the nodes of the tree with pathological elements of the second group are excluded. As for grounded trees with pathological elements of the first group, distribution of pathological elements in edges of the tree, is not important.

D. Algorithm of generation of equivalent trees

We now propose an algorithm to generate equivalent trees with all types of pathological elements. Initial conditions for the algorithm below is a tree of elements from first group: nullator and/or VM, or second: norator and/or CM-group.

1. *If this tree does have a grounded node*:

1.1. Generate all trees based on these nodes.

1.2. Each edge in these trees can be either nullor or mirror element, so all combinations of types of the edges should be made for each trees. Exit.

2. *If this tree does not have a grounded node:*

2.1. *If tree does not have VM (CM)*, then generate all trees based on these nodes and select all edges of the trees as nullators (norators). Exit.

2.2. *If a tree has VM (CM):*

2.2.1. Assign tag "+" to the arbitrary node of the tree.

2.2.2. For all edges of this tree, which are incident to the selected node, tag the second node according to the type of the edge (Table I)

2.2.3. Repeat step 2.2.2 until all nodes of the tree are tagged.

2.2.4. Generate all trees based on these nodes, preserving their tags; edges of these trees are considered uncertain.

2.2.5. For each tree determine types of the uncertain edges according to the Table I. Exit.

A number of equivalent trees in case of grounded tree is equal to the number of trees multiplied by 2 to the power of the number of edges. For the floating tree, a number of equivalent trees is less and is equal to the number of trees constructed on the corresponding nodes [11].

The algorithm validity is corroborating by Table I proved above. The algorithm can be used to generate new equivalent circuits with different trees of pathological elements.

III. Illustrative Example

As an example of equivalent transformations we will consider circuit of universal filter based on three ICCII+ proposed in [13], [14]. A model of such a filter with mirror elements is shown in Fig. 9.

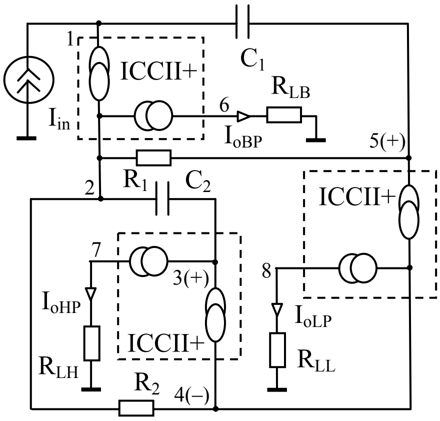

Fig. 9. Universal filter circuit based on three ICCII+.

Two VM form a tree on nodes 3, 4 and 5. Applying the rule of tagging, nodes 3 and 5 will have the tag (+) and node 4 will have the tag (–). This tree is equivalent to the tree shown in Fig. 4(a). Having built two other trees corresponding to Fig. 4 one can get two new equivalent circuits, in which one of VM is replaced by nullator as shown in Fig. 10 and Fig. 11.

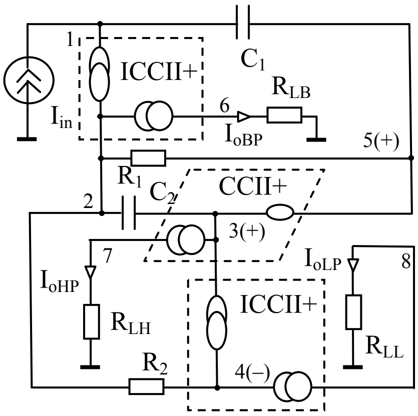

Fig. 10. The first equivalent circuit (for the circuit on Fig. 9) based on two ICCII+ and a CCII+, in which one of VM is replaced by nullator.

When combining (pairing) nullator with one of the CM, we will get the equivalent circuits based on two ICCII+ and a CCII+. Note that the circuit on Fig. 11 also illustrates a known method of different pairing of pathological elements. Equivalence of the obtained circuits confirmed by equivalence of symbolic analysis results.

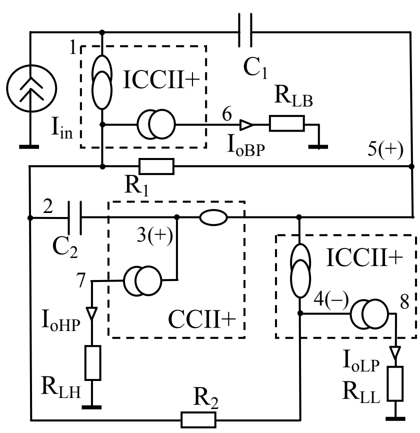

Fig. 11. The second equivalent circuit (for the circuit on Fig. 9) based on two ICCII+ and a CCII+, in which one of VM is replaced by nullator.

IV. Conclusions

The generalization and proof of equivalent transformations of trees with pathological elements is presented in the paper. The algorithm of synthesis of circuit with mirror and nullor elements based on tagging of nodes of the initial tree of these elements and selecting new pathological elements is proposed.

References

[1] J. Braun, “Topological analysis of networks containing nullators and norators,” Electron. letters, vol. 2, no. 11, pp. 427–428, 1966.

[2] L. T. Bruton, RC active circuits: theory and design. Englewood Cliffs, NJ: Prentice-Hall, 1980, pp.523.

[3] G. M. Wierzba, “Op-Amp Relocation: A Topological Active Network Synthesis,” IEEE Trans. on circuits and systems, vol. CAS-33, no. 5, pp. 469–475, May 1986.

[4] D. G. Haigh, “Analytic approach to nullor transformations for FET circuit synthesis: Part I – nullator-norator tree transformations,” in IEEE Proc. ISCAS, pp. 5231–5234, 2006.

[5] K. Anuntahirunrat, C. Fongsamut, W. Surakampontorn, N. Fujii, “On the Use of Norator Transformation in Oscillator Circuits,” In Proc. of IEEE Region 10 Conference TENCON-2006, Hong Kong, China, pp. 1 – 5, Nov. 2006.

[6] H. Y. Wang, S. H. Chang, Y. L. Jeang and C. Y. Huang, “Rearrangement of mirror elements,” Analog integrated circuits and signal processing, vol. 49, no.1, pp. 87–90, Jan. 2006.

[7] H. Y. Wang, N. H. Chiang, Q.M. Nguyen and S. H. Chang. "Advanced Materials, chapter 26: Circuit Synthesis Using Pathological Elements", Springer Proceedings in Physics, vol. 152, pp 317-328, 2014.

[8] A. M. Soliman and R. A. Saad, “The voltage mirror-current mirror pair as a universal element,” Int. J. circuit theory Appl, vol. 38, no. 8, pp. 787–795, 2010.

[9] C. Sanchez-Lopez, F. V. Fernandez, E. Tlelo-Cuautle and S. X.-D. Tan, “Pathological element-based active device models and their application to symbolic analysis,” IEEE Trans. on circuits and systems-I: regular papers, vol. 58, no. 6, pp. 1382–1395, Jun. 2011.

[10] V. V. Filaretov and K.S. Gorshkov, “Topological analysis of active networks containing pathological mirror elements,” In Proc. of IEEE XXXIII International Scientific Conference "Electronics and nanotechnology" (ELNANO–2013), Kiev, Ukrain, pp. 293–296, April 2013.

[11] N. Christofides, Graph Theory: An Algorithmic Approach. New York, London, San Francisco: Academic Press, 1975, pp.400.

[12] A. C. Davies, “Matrix Analysis of Networks Containing Nullators and Norators,” Electron. Letters, vol. 2, no. 2, pp. 48–49, Feb.1965.

[13] H. Y. Wang, C. T. Lee and C. Y. Huang, “Characteristic Investigation of New Pathological Elements,” Analog integrated circuits and signal processing, vol. 44, no. 1, pp. 95–102, Jan.2005.

[14] S. Ozoguz, A. Toker and O. Cicekoglu, “First-order allpass sections-based current mode universal filter using ICCIls,” Electron. Letters, vol. 36, no. 17, pp. 1443–1444, Aug. 2000.