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Research Article

The Generalization of the Extra Element Theorem for Symbolic Circuit Tolerance Analysis

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A new method of the symbolic circuit tolerance analysis has been proposed. The approach is based on Middlebrook's extra element theorem and the generalized parameter extraction method. It does not need the matrix network description or presentation as algebraic sets or topological graphs. The proposed techniques have been realized in the computer program (Toleralize). To verify the theoretical analysis, computer simulation results are included.

1. Introduction

The transfer function of the linear time-invariant network is generally presented as a rational fraction. But in many cases, especially in network synthesis, it is more appropriate to write the transfer function as follows [1]:

$$S = S_0(1+\gamma), \tag{1}$$

where S_0 is a nominal transfer coefficient, and γ is a multiplicative relative error. Both the variables S_0 and γ can be presented as a rational fraction.

The transfer function in the form (1) can be obtained by the usage of the Middlebrook's extra element theorem [1–3]

$$S = S_0\left(\frac{1+Z_{n1}/\chi}{1+Z_{d1}/\chi}\right),\qquad S = S_0'\left(\frac{1+\chi/Z_{n1}}{1+\chi/Z_{d1}}\right), \tag{2}$$

where χ is a parameter of an arbitrary circuit element (impedance, conductance, and controlled source), S_0 and S_0' are nominal transfer functions of the initial circuit in which $\chi \to \infty$ or $\chi \to 0$ correspondingly, Z_{n1} is a driving-point impedance of the network from the port where χ^-element is connected, with the input port open, and Z_{d1} is the driving-point impedance of the network at the port where χ-element is connected, with the input port short.

In case of $\chi \to \infty$, the selected element must be omitted from the circuit if χ is an impedance and replaced by a short or by a nullor [4–6] if χ is a conductance or a controlled source correspondingly. In case of $\chi \to 0$, the selected element must be replaced by a short if χ is an impedance, and deleted if χ is a conductance or a controlled source.

So, the usage of formulae (2) required computation of three different circuit functions Z_{n1}, Z_{d1} and S_0 or S_0'. This requirement makes problems for the symbolic circuit analysis of complicated network.

In Section 2 of this paper, the technique for improvement of the estimation efficiency of the extra element theorem is proposed. The obtained results provide the development of new symbolic circuit tolerance analysis method presented in Section 3. A practical example is discussed in Section 4. The conclusion is finally drawn in Section 5.

2. Extra Element Theorem and Generalized Parameter Extraction Method

In accordance with [7], the transfer function of electronic network can be expressed as

$$S = \frac{N}{D}, \tag{3}$$

where N is the determinant of the network, in which the independent source and arbitrary response are replaced by nullor, and D is the determinant of the network, in which the independent excitation and the arbitrary response are zero.

The calculation procedure of the network determinants is based on the recursive usage of the parameter extraction formula which generalizes the known Feussner's equations [8, 9]

$$\Delta = \chi\Delta\left(\chi \longrightarrow \infty\right) + \Delta\left(\chi = 0\right), \tag{4}$$

where χ is a parameter of arbitrary circuit element, $\Delta(\chi \to \infty)$ and $\Delta(\chi \to 0)$ correspond to determinants of the circuit matrix in which the parameter of extracted elements $\chi \to \infty$ or $\chi \to 0$, respectively.

The described techniques develop the singular elements approach discussed by Braun [7], Parten and Seacat [10], and Hashemian [11] and have been realized in the generalized parameter extraction method, an effective tool for network symbolic analysis [12–14] and synthesis [15]. The method can be used for the analysis of the circuit with all types of controlled sources. It requires neither matrix, nor ordinary graph description of the circuit. The main advantage of the method is that there are no cancellations among the generated terms.

The generalized parameter extraction method has been realized in the program CIRSYM as a part of the software tool SCAD (http://intersyn.narod.ru/scad.htm).

So, it is possible to present the formulae (2) with the help of network determinants as

$$S = \frac{N^{\chi}}{D^{\chi}}\left(\frac{\chi + N_{\chi}/N^{\chi}}{\chi + D_{\chi}/D^{\chi}}\right), \tag{5}$$

$$S = \frac{N_{\chi}}{D_{\chi}}\left(\frac{1 + \chi\left(N^{\chi}/N_{\chi}\right)}{1 + \chi\left(D^{\chi}/D_{\chi}\right)}\right), \tag{6}$$

where N^{χ} and D^{χ} are determinants of networks, in which the extracted parameter $\chi \to \infty$; N_{χ} and D_{χ} are determinants of networks, in which the extracted parameter $\chi \to 0$.

As evident from the expressions (5) and (6), we have two pairs of identical network determinants. So, for estimation of the transfer function in the form (1), we need to obtain only four network determinants: N^{χ}, D^{χ}, N_{χ}, and D_{χ}. It is simpler than a separate computation of circuit functions from (2): $S_0 = N^{\chi}/D_{\chi}$ or $S_0' = N_{\chi}/D^{\chi}$, $Z_{n1} = N_{\chi}/N^{\chi}$, and $Z_{d1} = D_{\chi}/D^{\chi}$. So, the proposed technique provides more than 30% increase of the computation efficiency.

In the case of a multiple extraction, that is, a simultaneous extraction of several parameters $\chi_1, \chi_2, \ldots, \chi_n$, it is possible to modify the formulae (5) and (6) as follows:

$$\begin{aligned} S &= S_0 \frac{\chi_1\left(N^{\chi_1}_{\chi_{2,\ldots,n}}/N^{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_n\left(N^{\chi_n}_{\chi_{1,2,\ldots,n-1}}/N^{\chi_{1,2,\ldots,n}}\right) + \chi_1\chi_2\left(N^{\chi_{1,2}}_{\chi_{3,\ldots,n}}/N^{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_1\chi_2\cdots\chi_n + N^{\chi(1,2,\ldots,n)}_{\chi(1,2,\ldots,n)}/N^{\chi_{1,2,\ldots,n}}}{\chi_1\left(D^{\chi_1}_{\chi_{2,\ldots,n}}/D^{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_n\left(D^{\chi_n}_{\chi_{1,2,\ldots,n-1}}/D^{\chi_{1,2,\ldots,n}}\right) + \chi_1\chi_2\left(D^{\chi_{1,2}}_{\chi_{2,\ldots,n}}/D^{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_1\chi_2\cdots\chi_n + D^{\chi(1,2,\ldots,n)}_{\chi(1,2,\ldots,n)}/D^{\chi_{1,2,\ldots,n}}}, \\ S &= S_0' \frac{1 + \chi_1\left(N^{\chi_1}_{\chi_{2,\ldots,n}}/N_{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_n\left(N^{\chi_n}_{\chi_{1,2,\ldots,n-1}}/N_{\chi_{1,2,\ldots,n}}\right) + \chi_1\chi_2\left(N^{\chi_{1,2}}_{\chi_{3,\ldots,n}}/N_{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_1\chi_2\cdots\chi_n\left(N^{\chi(1,2,\ldots,n)}_{\chi(1,2,\ldots,n)}/N_{\chi_{1,2,\ldots,n}}\right)}{1 + \chi_1\left(D^{\chi_1}_{\chi_{2,\ldots,n}}/D_{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_n\left(D^{\chi_n}_{\chi_{1,2,\ldots,n-1}}/D_{\chi_{1,2,\ldots,n}}\right) + \chi_1\chi_2\left(D^{\chi_{1,2}}_{\chi_{2,\ldots,n}}/D_{\chi_{1,2,\ldots,n}}\right) + \cdots + \chi_1\chi_2\cdots\chi_n\left(D^{\chi(1,2,\ldots,n)}_{\chi(1,2,\ldots,n)}/D_{\chi_{1,2,\ldots,n}}\right)}, \end{aligned} \tag{7}$$

where $S_0 = N^{\chi_{1,2,\ldots,n}}/D^{\chi_{1,2,\ldots,n}}$ and $S_0' = N_{\chi_{1,2,\ldots,n}}/D_{\chi_{1,2,\ldots,n}}$.

3. Symbolic Circuit Tolerance Analysis

The network multiplicative relative error from (1) has a direct connection with the parameters spread of the circuit elements. In accordance with [16], the influence of the elements tolerances to the network characteristics can be simulated by including additional elements into the network. If we consider the additional elements as "extra" elements, it is possible to use the extra element theorem for the network tolerance analysis.

The tolerance $\delta(\chi_i)$ of the two-port circuit element χ_i can be simulated by series resistances or by shunt admittances as shown in Figures 1(a) and 1(b) correspondingly. The circuit simulation of the controlled sources tolerances is presented in Figures 1(c)–1(f).

The symbolic expression of the network multiplicative relative error corresponding to the notations of circuit functions (1) and (4) can be defined as

$$\gamma = \frac{\delta\left(\chi\right)\left(N^{\delta(\chi)}D_{\delta(\chi)} - N_{\delta(\chi)}D^{\delta(\chi)}\right)}{\delta\left(\chi\right)N_{\delta(\chi)}D^{\delta(\chi)} + N_{\delta(\chi)}D_{\delta(\chi)}}. \tag{8}$$

The fractional formulae for obtaining the tolerance of the arbitrary circuit elements $\delta(\chi_i)$ can be expressed in the same way

$$\delta\left(\chi\right) = \frac{\gamma N_{\delta(\chi)}D_{\delta(\chi)}}{N^{\delta(\chi)}D_{\delta(\chi)} - N_{\delta(\chi)}D^{\delta(\chi)}\left(\gamma + 1\right)}. \tag{9}$$

It is possible to generalize the expression (8) in case when multiple tolerance-simulated elements are added into a circuit simultaneously,

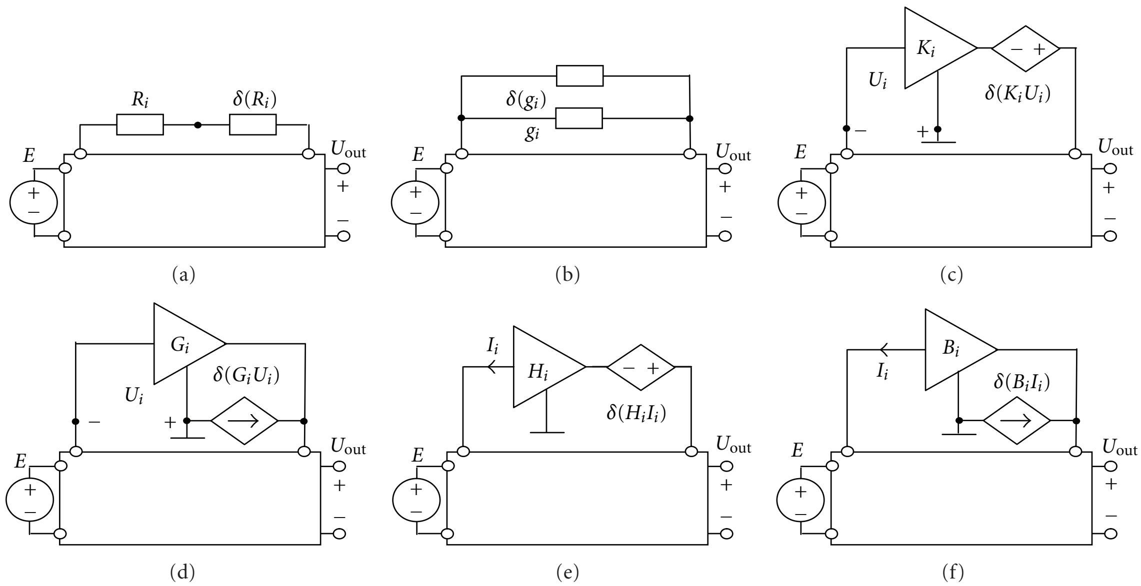

Figure 1: A circuit simulation of the elements tolerances.

$$
\begin{aligned}
&\gamma_{(1,2,\ldots,n)}\\
&\quad= \Big[\delta\big(\chi_1\big)\Big(N^{\delta(\chi_1)}_{\delta(\chi_{2,\ldots,n})}D_{\delta(\chi_{1,2,\ldots,n})} - N_{\delta(\chi_{1,2,\ldots,n})}D^{\delta(\chi_1)}_{\delta(\chi_{2,\ldots,n})}\Big) + \cdots + \delta\big(\chi_n\big)\Big(N^{\delta(\chi_n)}_{\delta(\chi_{1,2,\ldots,n-1})}D_{\delta(\chi_{1,2,\ldots,n-1})} - N_{\delta(\chi_{1,2,\ldots,n})}D^{\delta(\chi_1)}_{\delta(\chi_{1,2,\ldots,n-1})}\Big)\\
&\qquad+\delta\big(\chi_1\chi_2\big)\Big(N^{\delta(\chi_{1,2})}_{\delta(\chi_{3,\ldots,n})}D_{\delta(\chi_{1,2,\ldots,n})} - N_{\delta(\chi_{1,2,\ldots,n})}D^{\delta(\chi_{1,2})}_{\delta(\chi_{3,\ldots,n})}\Big) + \cdots + \delta\big(\chi_1\chi_2\cdots\chi_n\big)\Big(N^{\delta(\chi_{(1,2,\ldots,n)})}_{\delta(\chi_{(1,2,\ldots,n)})}D_{\delta(\chi_{1,2,\ldots,n})} - N_{\delta(\chi_{1,2,\ldots,n})}D^{\delta(\chi_{(1,2,\ldots,n)})}_{\delta(\chi_{(1,2,\ldots,n)})}\Big)\Big]\\
&\qquad\Big/\Big[N_{\delta(\chi_{1,2,\ldots,n})}\Big(\delta\big(\chi_1\big)D^{\delta(\chi_1)}_{\delta(\chi_{2,\ldots,n})} + \cdots + \delta\big(\chi_n\big)D^{\delta(\chi_n)}_{\delta(\chi_{1,2,\ldots,n-1})}\Big) + \delta\big(\chi_1\chi_2\big)D^{\delta(\chi_{1,2})}_{\delta(\chi_{3,\ldots,n})} + \cdots + \delta\big(\chi_1\chi_2\cdots\chi_n\big)D^{\delta(\chi_{(1,2,\ldots,n)})}_{\delta(\chi_{(1,2,\ldots,n)})} + D_{\delta(\chi_{1,2,\ldots,n})}\Big]
\end{aligned}
\tag{10}
$$

Note that the value of network determinants N_χ and D_χ in formulae (8) and (9) as well as determinants $N_{\chi_{1,2,\ldots,n}}$и$D_{\chi_{1,2,\ldots,n}}$ in (10) is invariable and can be found in the stage of nominal transfer function estimation.

The proposed techniques of tolerance analysis have been realized in the computer program Toleralize as a part of the software tool SCAD. Toleralize provides the estimation of symbolic expressions of a network transfer function in forms (5) or (6), a network multiplicative relative error in forms (8) or (10), and an element tolerance in form (9), as well as their numerical values.

4. Example

As an example, let us consider the procedure of the tolerance analysis of the transistor amplifier shown in Figure 2 [16]. The elements parameters are presented in Table 1 (first column).

It is possible to use the original formulae (2) for the tolerance analysis, but the technique on the base of network determinants approach considered in Section 3 makes the calculation process much simpler and faster.

First, we compute the nominal transfer function of the network by means of a generalized parameter extraction method

$$
\begin{aligned}
N_{\delta(\chi_i)} &= R_6(((R_3+R_4))(R_2(B_1+1))\\
&\qquad +(-B_2R_3)(-B_1(R_1+R_2)))\\
&= 101020200000000,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
D_{\delta(\chi_i)} &= (R_3+R_4)(-B_1R_1R_2+R_1(R_2+R_5)+R_2R_5)\\
&\quad +((R_3+R_4)R_6)(-B_1R_2+R_2+R_5)\\
&\quad +(R_3R_6B_2)(R_2B_1) = 1244400000000.
\end{aligned}
\tag{12}
$$

So, the nominal transfer function of the amplifier is $S_0 = 81, 18$.

Then, we use formula (8) for the estimation of network multiplicative relative errors provided that the elements parameters spread is $\delta(\chi_i) = \pm 10\%$. Note that we already know the value of network determinants $N_{\delta(\chi_i)}$ (11) and $D_{\delta(\chi_i)}$

Table 1: The elements parameters of the transistor amplifier shown in Figure 2 and the results of tolerance analysis.

χ_i	Symbolic expressions of network determinants N^χ and D^χ	γ%		$\delta(\chi_i)$%	
		$\delta(\chi_i) = +10\%$	$\delta(\chi_i) = -10\%$	$\gamma = +15\%$	$\gamma = -15\%$
R_1 10 kΩ	$N^\chi = R_6B_2R_3B_1 = 10000000000$; $D^\chi = (B_1R_2 + R_2 + R_5)(R_3 + R_4) = 22200000$	8	−8.3	19	−18
R_2 0.1 kΩ	$N^\chi = R_6((R_3 + R_4)(B_1 + 1) + (-B_2R_3)(-B_1)) = 10202000000$; $D^\chi = (R_3+R_4)(B_1R_1+R_1+R_5)+((R_3+R_4)R_6)(B_1+1) +(-R_3R_6B_2)(-B_1) = 12224000000$	−8.9	10.8	−13.4	18
R_3 1 kΩ	$N^\chi = R_6((R_2(B_1 + 1)) + (-B_2)(-B_1(R_1 + R_2))) = 101010100000$; $D^\chi = (B_1R_1R_2 + R_1(R_2 + R_5) + R_2R_5) +(R_6)(B_1R_2 + R_2 + R_5) + (-R_6B_2)(-R_2B_1) = 1122200000$	0.89	−1.1	−403	−64
R_4 1 kΩ	$N^\chi = R_6R_2(B_1 + 1) = 10100000$; $D^\chi = B_1(R_1 + R_6)R_2 + (R_1 + R_6)(R_2 + R_5) + R_2R_5 = 122200000$	−0.97	0.99	−133	180
R_5 1 kΩ	$N^\chi = 0$; $D^\chi = (R_1 + R_2 + R_6)((R_3 + R_4)) = 22200000$	−0.17	0.17	−731	989
R_6 1 kΩ	$N^\chi = (R_3+R_4)(R_2(B_1+1))+(-B_2R_3)(-B_1(R_1+R_2)) = 101020200000$; $D^\chi = (R_3 + R_4)(B_1R_2 + R_2 + R_5) + (-B_2R_3)(-R_2B_1) = 1022200000$	1.7	−1.9	271	−49.7
B_1 100 S	$N^\chi = R_6(B_2R_3(R_1 + R_2) + R_2(R_3 + R_4)) = 1010200000000$; $D^\chi = R_2(R_6B_2R_3 + (R_1 + R_6)(R_3 + R_4)) = 12200000000$	0.17	−0.21	−117	−90
B_2 100 S	$N^\chi = R_6R_3B_1(R_1 + R_2) = 1010000000000$; $D^\chi = R_6R_2B_1R_3 = 10000000000$	1.8	−2.1	198	−47

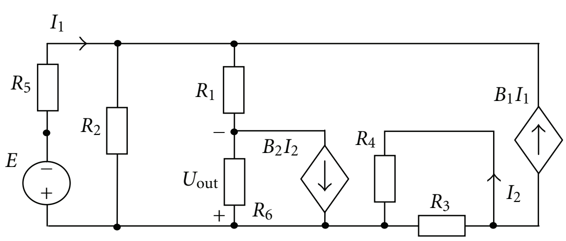

Figure 2: The linearized equivalent network of a transistor amplifier.

(12), so we need to obtain only two network determinants $N^{\delta(\chi_i)}$ and $D^{\delta(\chi_i)}$ for each χ_i circuit elements by means of a tolerances simulation approach. The results of the computation of the determinants symbolic expressions and their numerical values are presented in Table 1 (second and third columns, correspondingly).

Now, we estimate the elements tolerances provided that the multiplicative relative error of the amplifier transfer function is $\gamma = \pm 15\%$. The numeric values of network determinants included in expression (9) are already obtained, so the computation process becomes really simple. The calculated results of the parameters spread of circuit elements are presented in Table 1 (fourth column).

The results obtained have been verificated by the computer program Toleralize.

5. Conclusions

In this paper, a network determinant approach implemented for symbolic circuit analysis by means of the extra element theorem has been considered. The formulae for analytical evaluation of a network multiplicative relative error and the tolerance of the arbitrary circuit elements are presented. The proposed techniques provide more than 30% increase of estimation efficiency as compared to the original Middlebrook's method. The techniques proposed have been realized in the computer program Toleralize.

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