Generalized parameter extraction method in case of multiple excitation

Vladimir Filaretov* and Alexander Korotkov*

Abstract – **An extension of the generalized parameter extraction method for the case of multiple excitations is considered. The task of the analysis is to obtain the circuit response in symbolic form. The approach is based on simultaneous determinant calculation of numerator and denominator of the function that presents the response. An example of the method application illustrates the theoretical part of the paper.**

1 INTRODUCTION

Generalized parameter extraction method that was proposed and discussed in [1], [2] is an effective tool for network symbolic analysis. The method can be used for analysis of the circuit with all type of controlled sources. It does require neither matrix, nor ordinary graph description of the circuit. Analysis can be done using equivalent presentation of the circuit based on so called oriented nullor description of active elements. The symbol of the oriented nullor, firstly considered in [3], is shown in Fig.1

Figure 1: Oriented norator and nullator pair.

An idea of the method is expressed by the simple formula which generalizes known Feussner's equations (see references to Feussner's papers in [2], [4]) like as

$$\Delta = \chi\Delta(\chi \to 1) + \Delta(\chi \to 0)\,, \qquad (1)$$

where parameter χ corresponds to the value of the controlled source parameter, $\Delta(\chi \to 1)$ and $\Delta(\chi \to 0)$ correspond to determinants of the circuit matrix when the controlled source is replaced by the oriented nullor and when the controlled source is deleted from the circuit respectively. (It is necessary to note that the similar formula has been used in [5], but without proof of it). An illustration of the approach is in Fig.2, where all types of controlled sources are depicted. The calculated determinants are shown in the figure in circuitry–algebraic forms.

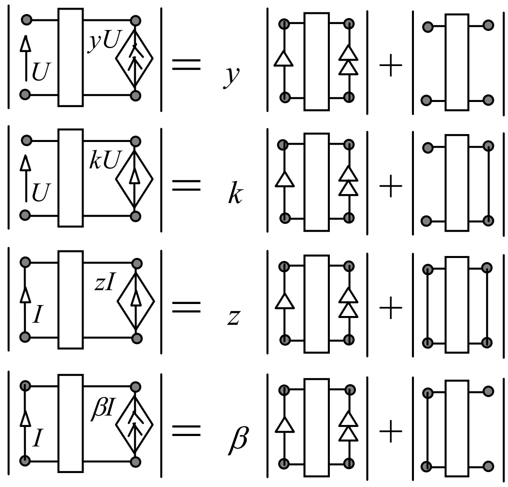


Figure 2: Extraction of the element parameters.

The calculation procedure has been described in details in previous publications [1], [2]. Thus, let us note here only principle statements. To calculate the determinant of the whole circuit eq.(1) is used together with nullor and passive element extraction formulas.

In each step of the determinant expansion the solvability conditions should be verified. General topological conditions for the solvability of linear networks can be formulated in the theorem proved in the paper [1] based on results proposed in [6], [7]. These necessary and sufficient conditions for the existence of solvability are

1. If and only if there is no loop consisting of both controlled-voltage-source branches and norators only nor no loop consisting of both current-sensor branches and nullator only.

2. If and only if there is no cutset consisting of both controlled-current-source branches and norators only nor no cutset consisting of both voltage-sensor branches and nullators only.

Symbolic methods are oriented mainly to generate expressions of the network functions. But in case of multiple excitations linear circuit is characterized by its output response. To calculate the response matrix determinants should be computed. It means that the parameter extraction method can be effectively used as well to solve the task. Thus, the purpose of current

* Department of Electrical Engineering, Ul'yanovsk State Technical University, Severnyi venets St., 32, Ul'yanovsk, Russia, 432027 e-mail: vvfil@mail.ru, tel.: +7 8422 381734.
* Department of Electrical Engineering and Telecommunications, St.Petersburg State Technical University, Polytechnic St. 29, St.Petersburg, Russia, 195251 e-mail: korotkov@rphf.spbstu.ru, tel.: +7 812 5527639.

paper is to extend an application of generalized parameter extraction method for the calculation of the network response in multiple excitation case.

The paper consists of four Sections. It is organized as follows. After the short Introduction an idea of the approach is described in Section II. Practical example is discussed in Section III. Conclusion summarizes the paper.

2 RESPONSE CALCULATION IN SYMBOLIC FORM

Let us consider the circuit under the influence of current and voltage sources J and E respectively as shown in Fig.3*a*. The task of the analysis is to calculate current I, which is indicated in the right branch of the circuit. This branch will be called further as a *current-sensor*. (Analogously the branch corresponding to the calculated or controlling voltage will be called as a *voltage-sensor*). To understand the conception of the approach the circuit of Fig.3*a* is equivalently transformed to the form of its augmented circuit (see Fig.3*b*) [4], where the independent sources are changed to their current controlled analogs. It is supposed that each source is controlled by own current I, which flows through the same branch.

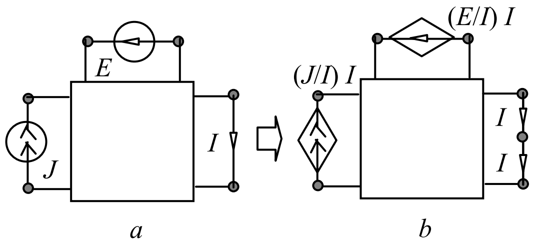


Figure 3: Equivalent transformation:
initial circuit (a), augmented circuit (b).

The calculated current is evaluated by means of the twice repeated expansion (1) due to generalized parameter extraction method. Thus the circuit determinant is expressed as a sum of three terms shown in Fig.4.

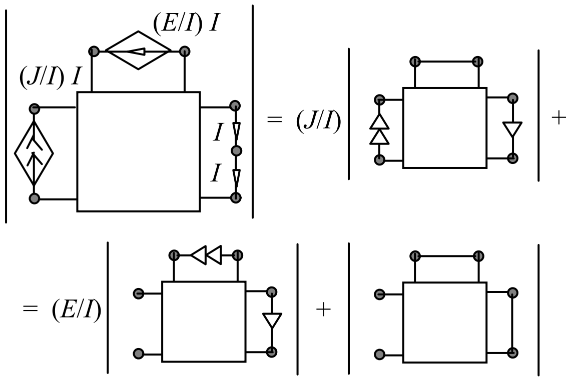


Figure 4: Expansion of the circuit determinant.

The augmented circuit does not include independent sources. Taking this fact into account and supposing that initially I is not equal to zero it is possible to conclude that determinant of this circuit is equal to zero. Say other words, the left side of circuitry–algebraic equation of Fig.4 is equal to 0. Thus, the current I and simultaneously output response of the circuit is determined by equation of Fig.5. The determinants in the numerator are calculated using generalized parameter extraction method.

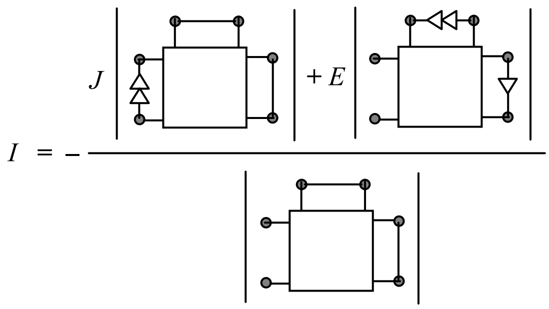


Figure 5: Circuitry-algebraic equation
for the calculation of the current response.

Let us note that current-sensor branch includes the nullator in each of the determinants of the numerator shown in Fig.5. This statement gives the possibility to introduce a new circuit element that can be formally called *nullator controlled multidimensional source*. Multidimensional source can consist of an arbitrary number of current and/or voltage sources. The controlling element is the same for each of the sources. It will be indicated in figures as a shaded nullator. Nullator of the source has got the same properties as a standard nullator. Thus, all known operations with nullators are still valid in this case. Obviously that the network can include only one nullator controlled multidimensional source. For the two-dimensional source that is discussed currently it is described analytically as

U=0, I=0

for the input nodes corresponding to the nullator and

U=E, I=J

for the output nodes corresponding to the sources. Parameters of initial independent sources E and J will be used as parameters of the nullator controlled multidimensional source.

Let us consider nullator controlled multidimensional source consisting of n current and voltage sources with parameters p_1, p_2, … p_n, where p_i is equal to E_i(or J_i). To extract parameters of multidimensional source p_i the equation (1) can be used in its modified form. On account of this reason the determinant Δ is expressed by the following formula

$$\Delta = p_i \Delta_1 + \Delta_2 . \qquad (2)$$

The network associated with Δ_1 is obtained from the initial network when the source with parameter p_i corresponding to E_i or J_i is replaced by a norator, the nullator of the multidimensional source is replaced by a standard nullator, and other sources with parameters p_1, p_2, $\ldots p_{i-1}$, $\ldots p_{i+1}$, $\ldots$ p_n are contracting if they belong to the class of voltage sources or are removing if they belong to the class of current sources. The network associated with Δ_2, is obtained from the initial network when the extracted voltage sources E_i are contracting or the extracted current sources J_i are removing. Δ_2 will be equal to zero if all parameters of *nullator controlled multidimensional source* have been extracted. Thus, the equation of Fig.5 can be represented as shown in Fig. 6.

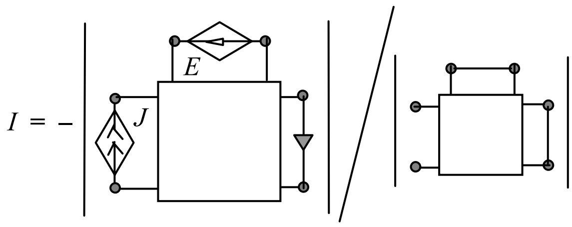


Figure 6: Circuitry-algebraic equation for evaluation of the current response.

The same approach is used when the voltage output response is calculated (see Fig.7).

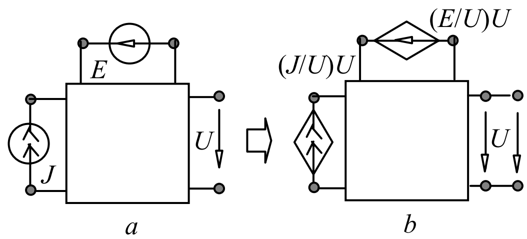


Figure 7: Equivalent transformation: initial circuit (a), augmented circuit (b).

Dropping intermediate calculation, the circuitry-algebraic equation for the evaluation of the voltage output response is presented in the form as it is shown in Fig.8.

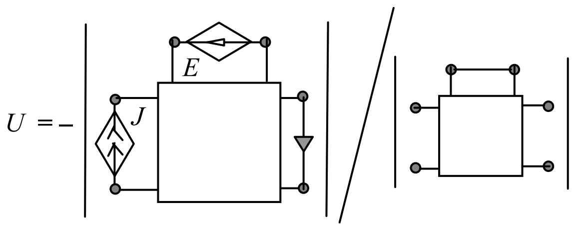


Figure 8: Circuitry-algebraic equation for the evaluation of the voltage response.

The discussed method generalizes the sorting scheme proposed in the paper [4]. The introduction of the new circuit element *nullator controlled multidimensional source* allows the calculation of the function numerator corresponding to the response as a unified equation. The approach has been realized in the program CIRSYMD as a part of the software tool SYMBOL. The program is free distributed. To obtain it, please, contact to vvfil@mail.ru. Numerical experiments using CIRSYMD have demonstrated that the proposed concept reduces a number of arithmetic operations (summation and multiplication) per 20-40% in comparison to approach based on superposition principle.

3 EXAMPLE

As an example of the method application we'd like to analyze the circuit shown in Fig. 9a. It consists of the differential amplifier terminated by a two-pole with the transfer function $T=N_2/D_2$. The augmented version of the circuit is presented in Fig.9b. The task of the analysis is to calculate the output voltage of the amplifier U in symbolic form like as $U=-N/D$. Let us note that the voltage-sensor corresponds to the voltage drop U. The calculation procedure is illustrated by Fig.10 and Fig.11.

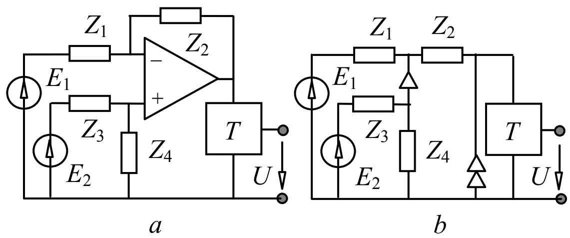
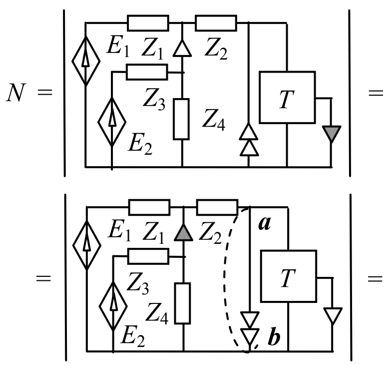
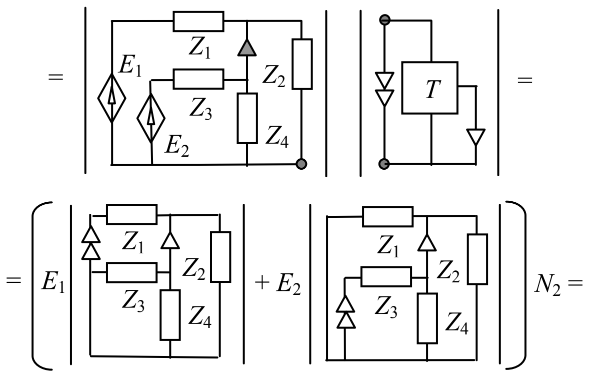


Figure 9: Differential amplifier (a), nullor based equivalent circuit (b).

To calculate numerator N of the response function the independent voltage sources E_1 and E_2 are represented as a nullator controlled two-dimensional voltage source where the controlling nullator is located at the output node instead of voltage-sensor.





$= N_2[E_1\, Z_2\, (Z_3{+}Z_4) - E_2\, Z_4\, (Z_1{+}Z_2)].$

Figure 10: Calculation of the function numerator.

Following operations are demonstrated in Fig.10: preliminary equivalent transformations when two nullators change places and orientation of the norator is changed as well (the determinant sign is preserved after these operations); the decomposition of the circuit in accordance to modification of bisection theorem for the case of section in the profile ***a-b*** [1]; the extraction of the *nullator controlled two-dimensional source* parameters in accordance to equation (2).

To calculate the function denominator excitation sources E_1, E_2 are contracted and the voltage-sensor U is removed from the equivalent circuit of the amplifier.

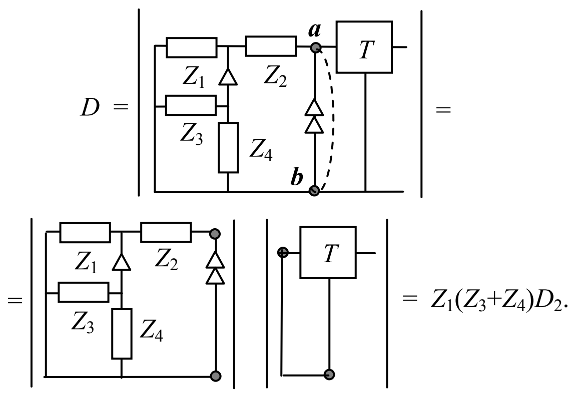


Figure 11: Calculation of the function denominator.

Following operations are demonstrated in Fig.11: the decomposition of the circuit in accordance to bisection theorem in the profile ***a-b*** [1]; the element Z_2 is contracted because it is connected in series with norator; the element Z_1 is extracted because it is connected in parallel with norator; the series connection of norator and nullator is removed.

Thus, the voltage response of the amplifier is expressed by the following formula

$$U = -\frac{N}{D} = \frac{E_2 Z_4 (Z_1 + Z_2) - E_1 Z_2 (Z_3 + Z_4)}{Z_1 (Z_3 + Z_4)} \cdot \frac{N_2}{D_2}. \quad (3)$$

As one can see, equation (3) corresponds to the transfer function of the cascaded connection of the differential amplifier and its terminal. If the approach based on superposition principle is used the result will be expressed as

$$U = \frac{E_2 Z_4 (Z_1 + Z_2) N_2 - E_1 Z_2 (Z_3 + Z_4) N_2}{Z_1 (Z_3 + Z_4) D_2}.$$

Thus, the number of operations in the last case will be increased by a number of operations which is needed to calculate N_2 and to multiply it in the second term of the numerator.

4 CONCLUSIONS

The proposed method allows the calculation of the current or voltage responses of the circuit in a case of multiple excitations in symbolic form. It is based on the conception of *nullator controlled multidimensional source* and does not require special calculation of partial transfer function from each of the input sources. The comparison with the standard approach based on superposition principle shows an advantage of the proposed method.

References

[1] V.V.Filaretov, "A topological analysis of electronic circuits by a parameter extraction method," *Electrical technology Russia*, no. 2, pp.46-61**,** 1998.

[2] V.V.Filaretov and A.S.Korotkov, "Generalized parameter extraction method in symbolic network analysis," in *Proc. ECCTD,* Kraków, Poland, vol. 2, pp. 406–409, Sept. 2003.

[3] J.Braun, "Topological analysis of networks containing nullators and norators," *Electronics letters*, vol.2, no. 11, pp. 427–428, Nov. 1966.

[4] G.E.Alderson and P.M.Lin, "Computer generation of symbolic network functions – a new theory and implementation," *IEEE Trans. Circuit Theory*, vol. CT-20, no. 1, pp. 48-56, Jan. 1973.

[5] R.Hashemian, "Symbolic representation of network transfer functions using norator-nullator pairs," *Electronic circuits and systems*, vol.1, no.6, pp.193–197, Nov. 1977.

[6] M.M. Milic, "General passive networks – solvability, degeneracies, and order of complexity," *IEEE Trans. Circuits and Systems*, vol. CAS-21, no.2, pp.177–183, March 1974.

[7] T.Ozawa, "Topological conditions for the solvability of linear active networks," *Circuit Theory and Applications*, vol.4, pp.125–136, 1976.