Topological Analysis of Active Networks Containing Pathological Mirror Elements

Vladimir Filaretov
Department of Electrical Engineering
Ul'yanovsk State Technical University
Ul'yanovsk, Russia
vvfil@mail.ru

Konstantin Gorshkov
Department of Electrical Engineering
Ul'yanovsk State Technical University
Ul'yanovsk, Russia
K.Gorshkov@ulstu.ru

***Abstract* — The new topological rules providing symbolic analysis of the networks with pathological mirror elements have been proposed. The approach is based on generalized parameter extraction method and it doesn't require presentation of the circuit in matrix or oriented graph form. The main advantage of this method is that there are no reductions of the generated terms of symbolic expressions. The method has been implemented in the computer program SCADS. A discussion of theoretical basics as well as results of the calculation is included.**

Keywords — symbolic analysis; nullor; voltage mirror; current mirror; pathological elements, network function.

I. INTRODUCTION

The pathological elements nullor [1-3] and voltage mirror-current mirror (VM-CM) pair [4] are quite useful in analog behavioral modelling and circuit synthesis [3, 5-7]. The nullor is an ideal operational amplifier which is composed of a nullator connected in the input-port and a norator connected in the output port. The grounded mirror elements VM and the CM make a pair which is also an ideal element and is composed of a VM at the input port and a CM at the output port.

The nullor-equivalents for the VM and CM are presented in Fig. 1 (a) and Fig. 1 (b) respectively [8]. If any terminal of the VM or CM is connected to the ground, it is equivalent to a nullator or norator element, respectively.

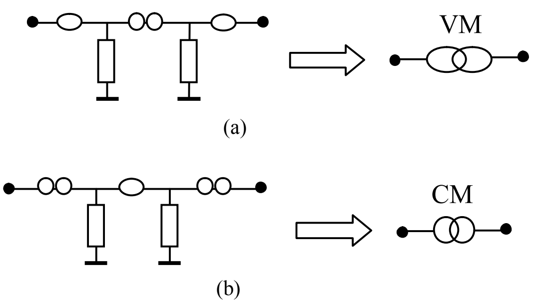

Fig. 1. The nullor-equivalents for the VM (a) and CM (b)

This models provide the implementation of the standard nodal analysis methods for calculation of the symbolic circuit functions of the active networks [8]. Also the construction of the nodal analysis matrix direct from networks containing the pathological mirror elements is possible as shown in [9]. Unfortunately, there is no report about usage of topological approach advantages of VM-CM based network analysis.

In this paper the special rules and the algorithm for symbolic analysis of active networks with pathological mirror elements are proposed. The approach is based on the generalized parameter extraction method, an effective tool for network symbolic analysis [10-12] and synthesis [13-15].

II. THE TOPOLOGICAL APPROACH TO SYMBOLIC ANALYSIS OF NETWORKS WITH PATHOLOGICAL ELEMENTS

A. *Orientation of Pathological Elements*

In contrast to symbolic analysis of nullor-based equivalent circuits by means of nodal analysis methods the implementation of topological approach requires the oriented nullor model presented in Fig. 2 [16]. Note, that the orientation of the nullator and norator corresponds to arrows shown as triangles instead of standard ovals [10].

Fig. 2. Oriented norator (a) and nullator (b)

The orientation of the nullors provide the direct formulation of circuit determinant by means of parameter extraction formulae without special presentation of the network as a number of algebraic sets or topological graphs [10]. If we use the oriented nullors for pathological mirror modelling we can obtain the oriented VM and CM as it shown in Fig. 3.

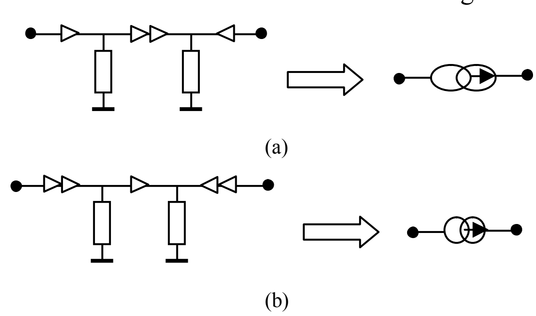

Fig. 3. Oriented VM (a) and CM (b)

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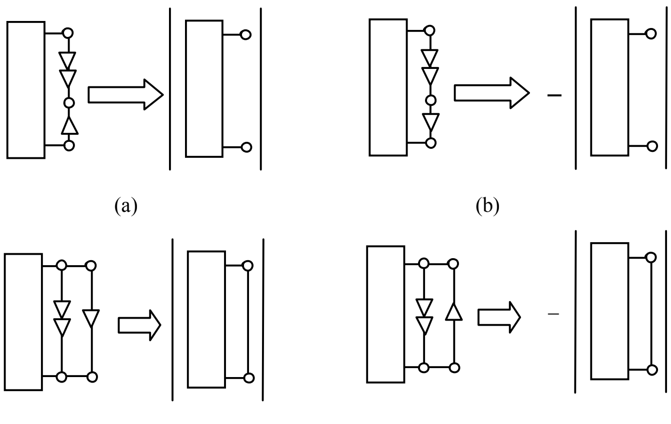
Let us suppose that the orientation of the VM (CM) corresponds to the middle norator (nullator) in the nullor equivalent network. The nullor elements from the right and left sides of the equivalent circuits in Fig. 3 are connected in opposite orientation.

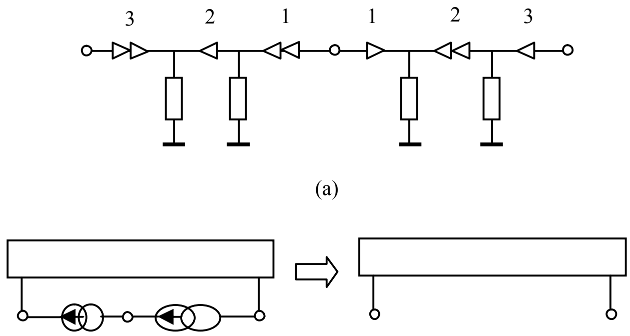
B. Equivalent Transformations of Pathological Elements-Based Circuits

The orientation of mirror elements provides the generalization of well-known topological rules of equivalent transformations of nullors-based circuits (Fig. 4) [10, 16, 17] for pathological mirror-based circuits. As shown in Fig. 5 (a) in case of series connection the VM-CM pair will be deleted from the circuit, because the nullor marked by index "1" corresponds to an open loop.



(a) (b) (c) (d)

Fig. 4. Equivalent transformations of nullors

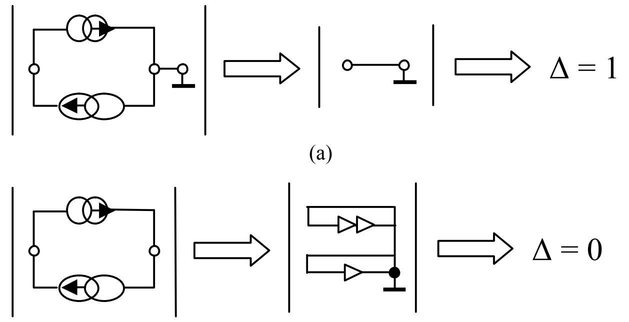


(a) (b)

Fig. 5. Transformation of the oriented VM-CM pair connected in series

It is obvious that the same thing will happen if we have a similar connection of the VM and norator or CM and nullator in series. Note, that the opposite connection of VM-CM, VM-nullator or CM-norator pair leads to the change of a determinant expression sign.

The transformation of mirrors in parallel connection will make sense only if one of their common nodes will be grounded (Fig. 6 (a)) or isolated (Fig. 6 (b)). In that case all of three nullors are deleted from the circuit and the mirrors loop is shorted. Note, that in case of a parallel connection the sign of the circuit determinant is still the same if the pathological elements have the different orientation and change if the orientation is similar.



(a) (b)

Fig. 6. Transformation of the oriented VM-CM pair connected in parallel

C. Special Cases of Circuit Elements Connection

It is very important for a circuit analysis by means of the generalized parameter extraction method that there are some special cases of elements connection providing sufficient simplicity of a network function calculation process. In addition to equivalent transformations of pathological elements considered above there are many topological rules that can be helpful for a symbolic analysis [11, 14, 19, 20]. They are presented in Table I.

TABLE I. THE CIRCUIT ELEMENTS IN SHORT-CIRCUIT AND IN OPEN LOOP

Element	Special Case	
	Short-circuit	***Open Loop***
Impedance	Parameter extracted – element deleted	Element shorted
Admittance	Element deleted	Parameter extracted – element shorted
Controlled voltage-source branch	Δ = 0	Element shorted
Current-sensor branch	Δ = 0	Element shorted
Controlled current-source branch	Element deleted	Δ = 0
Voltage-sensor branch	Element deleted	Δ = 0
Nullator	Δ = 0	Δ = 0
Norator	Δ = 0	Δ = 0
VM	Δ = 0	Δ = 0
CM	Δ = 0	Δ = 0

These special cases are the consequences of the implementation of the parameter extraction formula which generalizes the Feussner's equations [21, 22]:

$$\Delta = \chi\, \Delta(\chi \to \infty) + \Delta(\chi = 0), \qquad (1)$$

where χ is a parameter of arbitrary circuit element, $\Delta(\chi \to \infty)$ and $\Delta(\chi = 0)$ correspond to the determinants of the circuit matrix in which the parameter of extracted elements $\chi \to \infty$ or $\chi = 0$ respectively.

In case when $\chi \to \infty$ the selected element must be omitted from the circuit if χ is an impedance, and replaced by a short or by a nullor if χ is an admittance or a controlled source respectively. In case when $\chi = 0$ the selected element must be replaced by a short if χ is an impedance, and deleted if χ is an admittance. If the selected element is a controlled source (CS) then in case when $\chi = 0$ a voltage source and a controlling current must be replaced by a short, but a current source and a controlling voltage must be deleted.

In special cases presented in Table I one of the obtained determinants $\Delta(\chi\to\infty)$ or $\Delta(\chi = 0)$ will be equal to zero in accordance with the topological conditions for solvability of linear networks proposed in [11, 19, 20]. If the first summand of (1) is omitted from the expression the extracted parameter χ will be omitted too. If the second summand is omitted from the expression the extracted parameter χ will become a multiplier of a corresponding circuit determinant.

D. Pathological elements extraction

The procedure of nullor extraction can be formalized by the following steps [10]:

1. The choice of a supporting node. As a supporting node is the only node connected to singular elements can be chosen. The supporting node corresponds to the common node of a nullator and a norator of the extracted nullor.

2. The supporting node is splitted into two nodes for the first of them to be connected only with nullators while the second node must be connected only with norators.

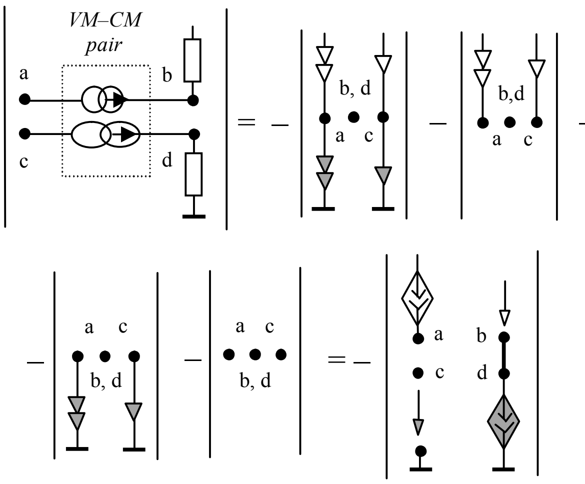
3. The branches corresponding to a nullator and norator of the extracted nullor are short-circuited.

The extraction rules of the nullor number n can be expressed by the following formula:

$$\Delta = \pm\, \Delta_n, \qquad (2)$$

where Δ_n is the determinant of the circuit after the procedure of the nullor number n extraction has been executed. Generally speaking, this equation means that the nullor extraction will change the sign of the initial determinant. The choice of the sign depends on the orientation of the nullor elements. If the norator and the nullator have the same orientation with respect to the supporting node the sign will be positive. In the opposite case the sign will be negative.

The generalization of formula (2) for the extraction of the pathological mirrors is illustrated below:



(3)

The meaning of the presented circuit transformation becomes clear if we compare it with the summation of two rows and two columns of a circuit matrix. It is obvious that these two procedures are isomorphic.

III. The Algorithm for a Symbolic Analysis of the Pathological Element-Based Circuits

In accordance with [16-18] the network function of a linear electronic circuit can be expressed as $\Delta N\ /\ \Delta D$, where ΔN is the determinant of the circuit, in which the independent source and an arbitrary response are replaced by a nullor, and ΔD is the determinant of the circuit, in which the input excitation and the arbitrary response are zero.

The procedure of the determinant calculation of the circuit with pathological elements is based on the following recursive algorithm:

1) equivalent transformation of two-ports elements (impedances in series and admittances in parallel);

2) search for special cases of elements connection (Table I);

3) equivalent transformation of the pathological elements in accordance with Fig. 4, Fig. 5 and Fig. 6;

4) extraction of nullors by means of formula (2);

5) extraction of pathological mirrors by means of formula (3);

6) parameter extraction by means of formula (1).

Note, that active networks must be checked by the rules presented in Table I after every change of circuit topology to avoid the violation of network solvability conditions.

The equivalent transformations of impedances in series and admittances in parallel must be done if possible to simplify the analysis process. The search for special cases of circuit elements connections (Table I) can simplify the calculation of circuit determinant even more. If circuit contain many pathological elements formulas (2) and (3) will be handy. Parameter extraction formula (1) must be used only when further topology simplicity is not possible. Eventually all parameters will be extracted from the initial network until we obtain one of the residual circuits presented in Fig. 7 [17].

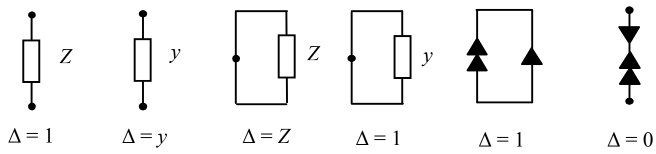


Fig. 7. The residual circuits and their determinants

Since the capacitors and the inductances can be presented as operational admittances and impedances respectively, so by means of the proposed method we can obtain the compact expression of the network function with a rational polynomial numerator and a denominator.

The approach discussed has been implemented in the program CIRSYM by V. Filaretov as a part of the software tool for the computer aided circuit design system SCADS. The demo-version of the program can be downloaded from the internet-site www.intersyn.narod.ru.

IV. Analysis Examples

To illustrate the proposed method, let us consider the symbolic analysis of the ICCII+-based inverting low-pass filter shown in Fig. 8 (a). The modified circuits for calculation of determinants of numerator ΔN and denominator ΔD of the network function $V_3/V_{in} = \Delta N / \Delta D$ are presented in Fig. 8 (b) and (c), respectively.

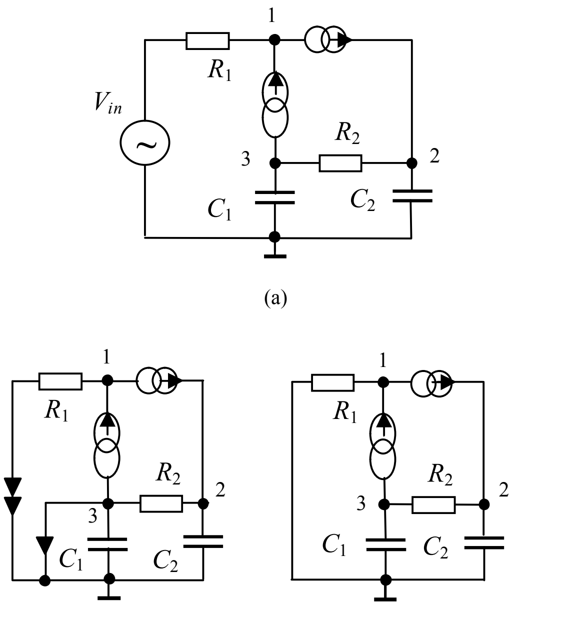


Fig. 8. Pathological element-based model of an inverting low-pass filter

The modified circuit for numerator3 calculation in Fig. 8 (b) will simplify because the topology including the resistor R_1 is connected in series with a norator and capacitor C_1 is connected in parallel with a nullator. In accordance with (1) and Table I a resistor must be shorted and capacitor must be deleted. Now we have to use the formula (1) for a parameter extraction of elements R_2 or C_2. In any case, both of the parameters will be omitted from determinant expression. Then we obtain the circuit consisting of only pathological elements. The determinant of this circuit can be easily calculated by means of a nullor extraction (3). The discussed procedures are illustrated by Fig. 9.

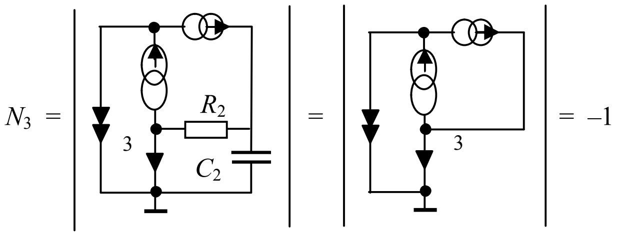
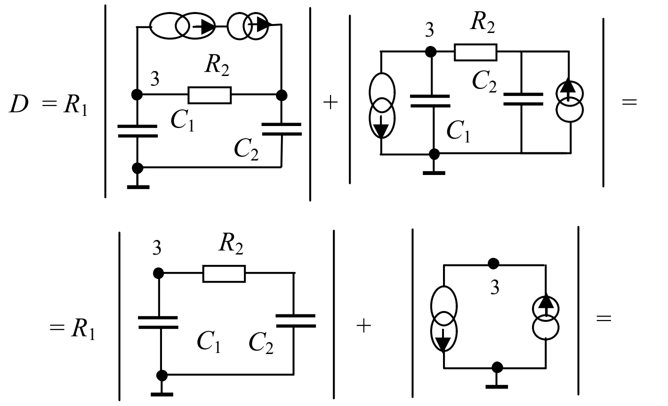


Fig. 9. The calculation of a numerator N_3

The extraction of R_1 from the modified circuit in Fig. 8 (c) leads to two subcircuits that satisfy the special cases of pathological elements connections. In the first one $R_1 \to \infty$ and the circuit can be simplified by deleting of VM-CM pair connected in series. The second circuit will include the operational admittance connected in parallel with pathological mirrors. After deleting of them VM and CM will be connected in series with R_2 and the circuit becomes a VM-CM pair grounded loop. So, the whole process of the denominator determinant calculation is illustrated by Fig. 10.



$$=R_1(R_2(p^2C_1C_2+p(C_1+C_2)))+1$$

Fig. 10. The calculation of a denominator D

The obtained expression of network function corresponding to the symbolic nodal analysis solution of the ICCII+-based inverting low-pass filter are presented in [8, 9]:

$$V_3/V_{in} = (-1)/(R_1(R_2(p^2C_1C_2+p(C_1+C_2)))+1). \quad (4)$$

The algorithm effectiveness has been tested by the analysis of the circuit presented in Fig. 11. The circuit consists of eight nodes and three VM–CM pairs.

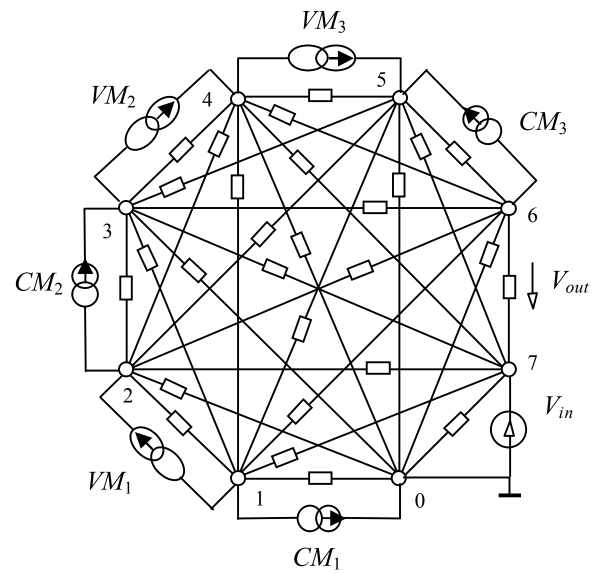


Fig. 11. The circuit for the analysis algorithm test

The circuit net-list in Spice-like format is presented in Table II. Note, that VM-CM pairs are marked by "M".

TABLE II. CIRCUIT NET-LIST

```
Vin   7  1
M1    1  0  1  2
M2    2  3  3  4
M3    6  5  4  5
g1    0  1  1
g2    0  2  2
….
g28 6  7  28
.end
```

The numeric results of the circuit analysis will be as follows:

$V_{out}=N/D = (-3521142) / 3144968= -1.1196113919124137.$

The calculation of V_{out} of circuit presented in Fig. 11 by means of a matrix approach done by doctor G. V. Mayko employing mathematic software Matlab leads to the same numeric results. The sufficient indexes of network function symbolic expressions obtained by different circuit analysis methods are presented in Table III.

TABLE III. EXPERIMENTAL RESULTS OF THE SCADS NETWORK FUNCTION GENERATION, COMPARED TO THE MATLAB

Indexes	Symbolic expressions calculated by	
	SCADS	Matlab
*	472	2383
+	951	700
–	182	189
(	726	0
)	726	0
number of terms	7794	11479

The proposed algorithm has been used for a symbolic analysis of many other pathological mirror-based networks. These analysis solutions can be downloaded from the internet-site www.intersyn.narod.ru.

V. CONCLUSIONS

The new topological method of network function calculation of networks with voltage mirror-current mirror has been proposed. The new equivalent transformations of pathological element-based network providing the simplicity of a symbolic circuit analysis are presented. The main advantage of the method is that it is cancellation free. There are no limitations related to the element types of linear networks. The proposed method has been implemented in the computer aided circuit design system SCADS.

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